

4. a) Calculate the probability that an electron in the ground state of hydrogen atom will be found within one Bohr radius of the nucleus.

$$\psi_{1s} = \frac{1}{(\pi a_0^3)^{1/2}} e^{-r/a_0} \quad [3]$$

- b) For a simple harmonic oscillator, zero point energy is the manifestation of Heisenberg's uncertainty relation. Explain. [2]

- c) Consider the following two wave-functions for the H-atom,

$$\psi_{2p+1} = b e^{-r/2a} \cdot r \sin \theta e^{i\phi};$$

$$\psi_{2p-1} = b e^{-r/2a} \cdot r \sin \theta e^{-i\phi}$$

where, a = Bohr radius, $b = \frac{1}{\sqrt{64\pi}} \left(\frac{1}{a}\right)^{5/2}$

Combine the above two functions to obtain the real ψ_{2p_x} function assuming that ψ_{2p+1} and ψ_{2p-1} are normalized and orthogonal to each other. Justify your answer mentioning an appropriate quantum mechanical theorem. [5]

Unit - III

5. a) The energy of 1 mol of solid assuming Einstein model is given by,

$$E = 3N_A \left(\frac{h\nu_E}{e^{h\nu_E/kT} - 1} \right), \text{ where terms have their usual significance.}$$

Find, (a) an expression for $\overline{C_v}$ as—

i) $T \rightarrow \infty$ ii) $T \rightarrow 0$ [4]

- b) Consider a system of a molecules, distributed among non-degenerate energy levels, $\epsilon_0, \epsilon_1, \epsilon_2 \dots$ etc .

Show that the internal energy (U) of the system can be expressed as $U = NK_B T^2 \left(\frac{\partial \ln Z}{\partial T} \right)_v$

[terms have usual meaning] [4]

- c) Consider a system of non-interacting particles at constant temperature which are distributed in 3 non-degenerate energy levels in such a way that ϵ_1, ϵ_2 and ϵ_3 consist of 4×10^{23} , 2×10^{23} and 1×10^{23} particles, respectively. Show that the energy levels are equispaced. [2]

6. a) Calculate the number of microstates of arranging 6 identical particles in two boxes, each having 5 compartments without any restriction on occupation such that 4 particles are present in one box and 2 particles are present in the second box. [3]

- b) Arrive at the form of Boltzmann distribution by applying the condition of $\ln W$ to be a maximum with other necessary constraints. [5]

- c) Define partition function and interpret it physically. [1+1]

Unit - IV

7. a) Blue $\text{CuSO}_4 \cdot 5\text{H}_2\text{O}$ crystals release their water of hydration when heated. How many phases and components are present in an otherwise empty heated container? [2]

- b) The specific volume of monoclinic sulphur which is stable above the transition temperature is greater than that of rhombic sulphur by $0.0126 \text{ cm}^3 \text{ g}^{-1}$. The transition point at one atm. pressure is 368.65 K and it increases at the rate of 0.035 K atm^{-1} . Calculate the molar heat of transition. [3]

- c) i) Sketch the phase diagram of water and mark the following in the diagram : Normal boiling pt., freezing pt., triple point and critical point. [1+2]

- ii) How does the phase diagram of water differ from that of carbon-di-oxide? Show by graph also. [2]

- d) Draw the phase diagram (T vs mole % of B) of a system consisting of solids A and B forming a stable compound A_2B with congruent melting point. Show that different phases present in the different regions of the diagram. State the degrees of freedom at eutectic point.
[M. Pt of $A_2B < M. Pt. of A < M.Pt. of B$] [5]
8. a) Derive thermodynamically a relation between the osmotic pressure of a dilute solution of a solute and its molar concentration. State assumptions and approximations involved. [4+1]
- b) Explain Konowaloff's rule. [2]
- c) Derive Gibbs-Duhem equation. [2]
- d) In an ideally dilute solution the solutes obey Henry's law. —Explain. [2]
- e) Consider a solute having normal existence in liquid α and associated in liquid β according to the equation, $nA \rightleftharpoons A_n$
- How will you determine the value of 'n' experimentally? [4]

